

A Magic Square for Plato

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In Plato's *Republic* is a celebrated passage pointing out the importance of the number 729. The passage (Book 9, 587–8 Lee's translation) reads:

'Do you know,' I asked, 'just how much unhappier the tyrant is than the philosopher king?'

'No, tell me.'.....

'You will find, if you work out the cube, that the measure of difference between the two in terms of true pleasure is that the philosopher king lives seven hundred and twenty-nine times more pleasantly than the tyrant, and the tyrant the same amount more painfully than the philosopher king.'

'What a terrific calculation,' he exclaimed, 'and all to show how much difference there is between the just and unjust man in terms of pleasure and pain!'

'But it's quite correct,' I replied, 'and fits human life, if human life is measured by days and nights and months and years.'

Nowadays many would consider it irrational to ascribe attributes to numbers, and would hold that the number 729 has no more significance in human affairs than the number 42 which, in *The Hitchhiker's Guide to the Galaxy* was given as the answer to Life, the Universe and Everything. However, in Plato's time mathematicians of immense stature, such as Pythagoras who said that number is the origin of all things, did indeed ascribe some ulterior significance, some almost supernatural or magical power to numbers. Plutarch set out the numbers attributed to the bodies in the solar system: Antichthon, Earth's doppelganger, is 3; Earth itself is 9; the Moon 27; Mercury 81; Venus 243 and the Sun 729.

Plato's (or more likely Pythagoras') arithmetic is plain to see – he arrives at the figure of 729 by squaring the cube of three – but it is less clear how he saw this number fitting into human life. Many explanations have been offered in the past. One suggestion long ago was that a 27 power square, (i.e. one of 27 x 27 cells) with the numbers 1 to 729 inserted in numerical order would reveal the number 365 as the central square, clearly indicating the days in a year. From this it was suggested that the number 27 indicates the days in the lunar month, with the number 729 indicating the number of days and nights in a calendar year. It was further suggested that if the square were set out on a black and white chequerboard, the days and nights would be self-evident.

This suggested explanation, though attempting to answer the question of days, nights and months, fails to account either for the years, or for the reason for increased happiness. If indeed a 27 power square was an essential step in arriving at Plato's conclusion then it must have been a more complex square than one set out in numerical sequence. Could it have been a magic square (i.e. a square where the rows, columns, and diagonals each total the same number)? The evidence would appear to be against it. The earliest written evidence of magic squares appears

130	86	152	128	88	150	135	84	145
144	102	118	139	104	125	137	106	123
110	160	96	117	156	91	112	158	98
85	149	134	83	151	132	90	147	127
108	120	136	103	122	143	101	124	141
155	97	114	162	93	109	157	95	116
148	131	89	146	133	87	153	129	82
126	138	100	121	140	107	119	142	105
92	115	159	99	111	154	94	113	161
535	491	557	533	493	555	540	489	550
549	507	523	544	509	530	542	511	528
515	565	501	522	561	496	517	563	503
490	554	539	488	556	537	495	552	532
513	525	541	508	527	548	506	529	546
560	502	519	567	498	514	562	500	521
553	536	494	551	538	492	558	534	487
531	543	505	526	545	512	524	547	510
497	520	564	504	516	559	499	518	566
454	410	476	452	412	474	459	408	469
468	426	442	463	428	449	461	430	447
434	484	420	441	480	415	436	482	422
409	473	458	407	475	456	414	471	451
432	444	460	427	446	467	425	448	465
479	421	438	486	417	433	481	419	440
472	455	413	470	457	411	477	453	406
450	462	424	445	464	431	443	466	429
416	439	483	423	435	478	418	437	485

653	719	695	655	717	702	651	712	292	248	314	290	250	312	297	246	307
669	685	706	671	692	704	673	690	306	264	280	301	266	287	299	268	285
727	663	684	723	658	679	725	665	272	322	258	279	318	253	274	320	260
716	701	650	718	699	657	714	694	247	311	296	245	313	294	252	309	289
687	703	670	689	710	668	691	708	270	282	298	265	284	305	263	286	303
664	681	729	660	676	724	662	683	317	259	276	324	255	271	319	257	278
698	656	713	700	654	720	696	649	310	293	251	308	295	249	315	291	244
705	667	688	707	674	686	709	672	288	300	262	283	302	269	281	304	267
682	726	666	678	721	661	680	728	254	277	321	261	273	316	256	275	323
329	395	371	331	393	378	327	388	211	167	233	209	169	231	216	165	226
345	361	382	347	368	380	349	366	225	183	199	220	185	206	218	187	204
403	339	360	399	334	355	401	341	191	241	177	198	237	172	193	239	179
392	377	326	394	375	333	390	370	166	230	215	164	232	213	171	228	208
363	379	346	365	386	344	367	384	189	201	217	184	203	224	182	205	222
340	357	405	336	352	400	338	359	236	178	195	243	174	190	238	176	197
374	332	389	376	330	396	372	325	229	212	170	227	214	168	234	210	163
381	343	364	383	350	362	385	348	207	219	181	202	221	188	200	223	186
358	402	342	354	397	337	356	404	173	196	240	180	192	235	175	194	242
5	71	47	7	69	54	3	64	616	572	638	614	574	636	621	570	631
21	37	58	23	44	56	25	42	630	588	604	625	590	611	623	592	609
79	15	36	75	10	31	77	17	596	646	582	603	642	577	598	644	584
68	53	2	70	51	9	66	46	571	635	620	569	637	618	576	633	613
39	55	22	41	62	20	43	60	594	606	622	589	608	629	587	610	627
16	33	81	12	28	76	14	35	641	583	600	648	579	595	643	581	602
50	8	65	52	6	72	48	1	634	617	575	632	619	573	639	615	568
57	19	40	59	26	38	61	24	612	624	586	607	626	593	605	628	591
34	78	18	30	73	13	32	80	578	601	645	585	597	640	580	599	647

in Chinese literature, written about A.D. 1100, though it is known that Arabian astrologers were using them to prepare horoscopes two or three centuries earlier. Although there is no evidence of magic squares in use in Ancient Greece, absence of evidence does not mean evidence of absence. It would have required no intellectual leap for a member of Pythagoras' school to configure the basic 3 power magic square, where the rows, columns and diagonals each total fifteen, as follows.

2	9	4
7	5	3
6	1	8

It seems likely that a school engrossed with numbers would have little difficulty in configuring such an elementary arrangement of numbers. Even so, it would still have required an extraordinary extrapolation of the basic square to produce a 27 power square with the necessary properties to justify Plato's statement.

Plato implied that to find the solution one should work out the cube. The cube root of 729 is 9, so it seems probable that he was implying that the key to the solution is the number 9. A 27 power square contains nine 9 power squares within its borders, and each of these squares, if they are part of the key, must have a significance beyond being part of the larger square. Did Pythagoras' school devise such a complex square, and if so, could it have been similar to the one I have constructed on pp. 30–31?

This square contains the numbers 1 to 729 in non-numerical order, with 365 occupying the centre square. It is a magic square, each row, column and diagonal totaling 9855. The square consists of 9 nine power magic squares, one containing the numbers 1 to 81, and the others with increments of 81 in succession. Each of these smaller squares has a numerical significance. The rows, columns and diagonals in each individual square produce a total, or constant, as follows:

Square 1369	1 year
2 1098	3 years
3 1827	5 years
4 2556	7 years
5 3285	9 years
6 4014	11 years
7 4743	13 years
8 5472	15 years
9 6201	17 years
Totals 29565	81 years

Each of the smaller squares within the large square thus indicates a certain age or period of time, and various combinations of these squares can indicate any period or age up to 81 years, a reasonable life span for an individual. These periods can themselves be set to form a 3 power squares, or, as shown in fig 3, a swastika.

3	17	7
13	9	5
11	1	15

The rows, columns and diagonals produce the constant 27, and the swastika, a symbol of well-being and happiness, common in Ancient Greek designs, totals 81.

From all the above one can derive the following facts, which to modern eyes are merely the product of simple arithmetic and the re-arrangement of the various powers of 3, but to Ancient Greek eyes could well have had a magic or supernatural significance. It would have been seen by those eyes that:

the centre square on p. 30–31 is occupied by the number 365 indicating the days in a year;

the number of days and nights is indicated by the number

729;

the 27 columns and rows show the number of days in a lunar month;

the constant of each of the smaller squares, either alone or in combination, can be developed to indicate a certain number of years up to and including 81;

the constant for each smaller square is produced by totalling not only the columns, rows and diagonals but also any nine numbers within its borders which together form a square;

the constant of the squares shown above is 27;

the number of days in 27 years is 9855, the constant of the larger square;

the total of the numbers in the swastika, the symbol associated with well-being and happiness, is 81;

the number 81 is based upon a 3 power square, so in a 27 power square it must be 81×9 which equals 729.

Thus, with the aid of creative exegesis, it could be suggested that fig. 2 fulfils Plato's requirements of fitting human life as measured by days, nights, months and years, and the 729 times pleasure enjoyed by the philosopher king.

There is an even more enigmatic paragraph in Plato's *Republic* (Book 8 546, Lee's translation) in which he sets out the right and wrong times for breeding. It reads:

*And though the Rulers you have trained for your city are wise, reason and observation will not always enable them to hit on the right and wrong times for breeding; some time they will miss them and then children will be begotten amiss. For the divine creature there is a period contained in a perfect number. For the human creature it is the smallest number in which certain multiplications, dominating and dominated, comprising three distances and four terms, give a final result, by making like and unlike, increasing and decreasing, which is commensurate and rational. Their basic ratio of four to three, coupled with five, and multiplied by three, yields two harmonies, of which one is the product of equal factors and of a hundred multiplied the same number of times, while the other is the product of factors of which some are equal, some unequal, that is, **either** a hundred squares of diagonal of rational number, each diminished by one, **or** a hundred squares of irrational number, each diminished by two, **and** one hundred cubes of three.*

No satisfactory solution has been given for this obscure paragraph, though many suggestions have been made in the past. Whatever the calculation, however, it is clear that the answer, however produced, is intended to indicate the period(s) within the life span of a man and/or a woman when it would be most appropriate for the begetting of children. It is no more than idle speculation, but if a magic square similar to that on pp. 30–31 were in being in ancient Greece, then, since it embraces periods, it could have provided a matrix for just such a calculation.

For those interested in magic squares, the nine smaller squares are pandiagonal. Any number of rows can be moved from top to bottom, and/or any number of columns can be moved from left to right, without destroying the constant produced by the diagonals.

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His interest in this problem originates from a class he attended on Plato's Republic. He admits that the conclusions which he reaches here are highly speculative.